THE THEORETICAL PREDICTION

OF THE

CENTER OF PRESSURE

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NARAM-8 R&D Project

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CENTER OF PRESSURE CALCULATIONS

Background

The most important characteristic of a model rocket is its stability.

A rocket's static stability is affected by the relative positions of its center of gravity(C.G.) and its center of pressure(C.P.). As is well known, the static margin of a rocket is the distance between the C.G. and C.P. A rocket is statically stable if the C.P. is behind the C.G.; also, it is more stable for a larger static margin.

The center of gravity of a rocket is easily determined by a simple balance test. The center of pressure, determination is much more difficult. Many methods for determining the C.P. have been proposed. The majority of them boil down to the determination of the center of lateral area which is the C.P. of the rocket if it were flying sideways. The center of lateral area is a conservative estimate; that is, it is forward of the actual C.P.; and, as such, is not a bad method for the beginner. However, as model rocketry becomes more sophisticated, and rocketeers become more concerned with reducing the static margin to the bare minimum; to reduce weather-cocking a more accurate method is called for.

The center of pressure is the furthest aft at zero angle of attack. By calculating the C.P. at $\alpha=0$; therefore; one has the least conservative value. It is this value to which any safety margin should be added.

The advantage of this method is that it reduces the static margin to a safe and pre-determined minimum.

The existance of an easily applicable set of equations for the calculation of the C.P. allows the rocketeer to truly design his

Background (cont.)

birds before any construction takes place. Since, by necessity, the derivation of any equations requires a predetermined configuration; a method of iteration must be used to determine the final design.

CENTER OF PRESSURE CALCULATIONS

Objective: To derive the subsonic theoretical center of pressure equations of a general rocket configuration. And to simplify the resulting equations without a great loss of accuracy so that the average leader can use them.

Method of Approach:

1. Divide rocket into separate portions.
2. Analyse each portion separatly.
3. Analyse the interference effects between the portions.
4. Simplify the calculations where necessary.
5. Recombine the results of the separate analyses to obtain the final answer.
6. Verify Analysis by experiment.

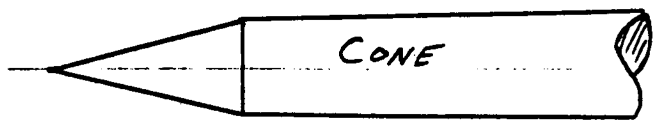
Assumptions:

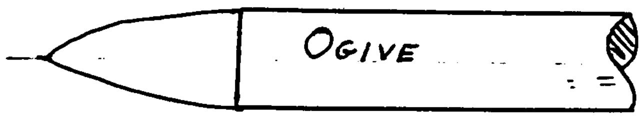
1. Flow over rocket is potential flow. ie, no vortices or friction
2. Point of the nose is sharp.
3. Fins are thin flat plates with no cant.
4. The angle of attack is very near zero.
5. The flow is steady state and subsonic.
6. The rocket is a rigid body.
7. The rocket is axially symmetric.

Portioning of Rocket

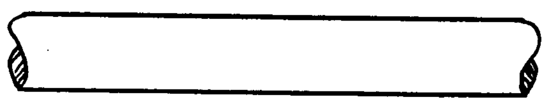
A rocket is, in general, composed of the following portions:

1. Nose



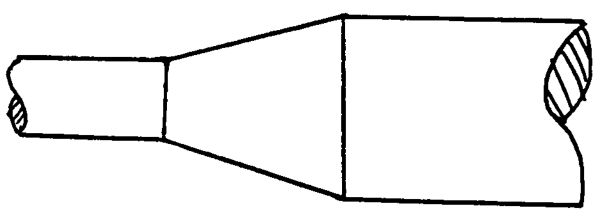


2. Cylindrical Body

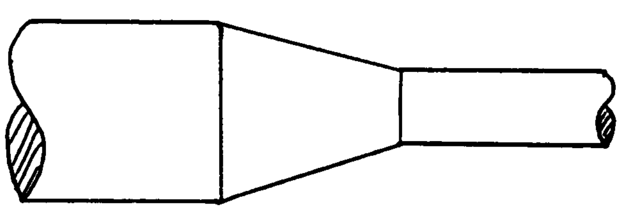


Different diameters before and after any conical sholder..

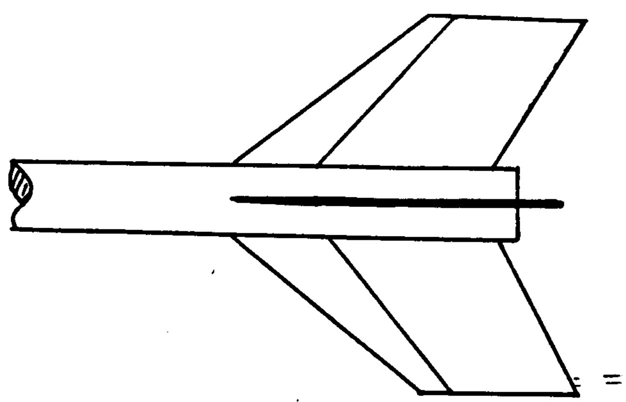
3. Conical sholder



4. Conical Boattail



5. Fins



Symbols

A = Reference Area = $\pi/4\, d^2$

A_f = Area of One fin

AR = Aspect Ratio

C = General Fin Chord Length

C_m = Nondimensional Aerodynamic Pitching Moment Coefficient = M/Aqd

C_{m_α} = Slope of Moment Coefficient Curve at $\alpha = 0$, $\partial C_m/\partial\alpha |_{\alpha=0}$

C_{MA} = Mean Aerodynamic Chord.

C_N = Nondimensional Aerodynamic Normal Force

C_{N_α} = Slope of the Force Coefficient at $\alpha = 0$, $\partial C_N/\partial\alpha|_{\alpha=0}$.

C_r = Root Chord Length

C_t = Tip Chord Length

d = Reference Length = Diameter at the base of the nose

F = Diederich's Correlation Parameter

f = Fineness Ratio

K = Interference Factor

L = Body Portion Length

ℓ = Length of Fin Midchord Line

$M(x)$ = Local Aerodynamic Pitching Moment About the Front of the Body Portion

$n(x)$ = Local Aerodynamic Normal Force

q = Dynamic Pressure = $\frac{1}{2}\rho V_0^2$

r_t = Radius of Body Between Fins

S = Fin Semispan

$S(x)$ = Local Crossectional Area

V_o = Free Stream Velocity

$\mathcal{V}$ = Body Portion Volume

$w(x)$ = Local Downwash Velocity

x = General Distance Along Body

$\bar{X}$ = Center of Pressure Location

X_F = Distance Between the Nose Tip and the Leading Edge of the Fin Root Chord

X_t = Distance Between the Root Chord Leading Edge and the Tip Chord Leading Edge in a Direction Parallel to the Body

y = General Fin Spanwise Coordinate

$\bar{Y}$ = Spanwise Location of Mean Aerodynamic Chord

α = Angle of Attack

Λ_L = Sweep of Fin Leading Edge

σ = Sweep of Fin Midchord Line

λ = Fin Taper Ratio = C_t/C_r

ρ = Freestream Density

Subscripts

B = Body

F = Tail or Fins

N = Nose

CS = Conical Shoulder

CB = Conical Boattail

$T(B)$ = Tail in the Presence of the Body

BODY AERODYNAMICS DERIVATIONS

Normal Force Coefficient Slope

General:

For an axially symmetric body of revolution; from reference 4; the subsonic steady state aerodynamic running normal load is given by;

$$n(x) = \rho V_0 \frac{\partial}{\partial x}\left[S(x)\, w(x)\right] \qquad (1)$$

where; (See figure 1)

- $n(x)$ = The running normal load per unit length.
- ρ = Free stream density
- V_0 = Free stream air speed
- $S(x)$ = Local crossectional area
- $w(x)$ = Local downwash at a given point on the body.

A rigid body has the downwash given by;

$$w(x) = V_0 \alpha \qquad (2)$$

Thus;

$$n(x) = \rho V_0^2 \alpha \frac{\partial S(x)}{\partial x} \qquad (3)$$

By the definition of normal force coefficient;

$$C_N(x) = \frac{n(x)}{q A} = \frac{n(x)}{\frac{1}{2}\rho V_0^2 A} \qquad (4)$$

Substituting equation 3 into 4

$$C_N(x) = 2 \frac{\alpha}{A} \frac{\partial S(x)}{\partial x} \qquad (5)$$

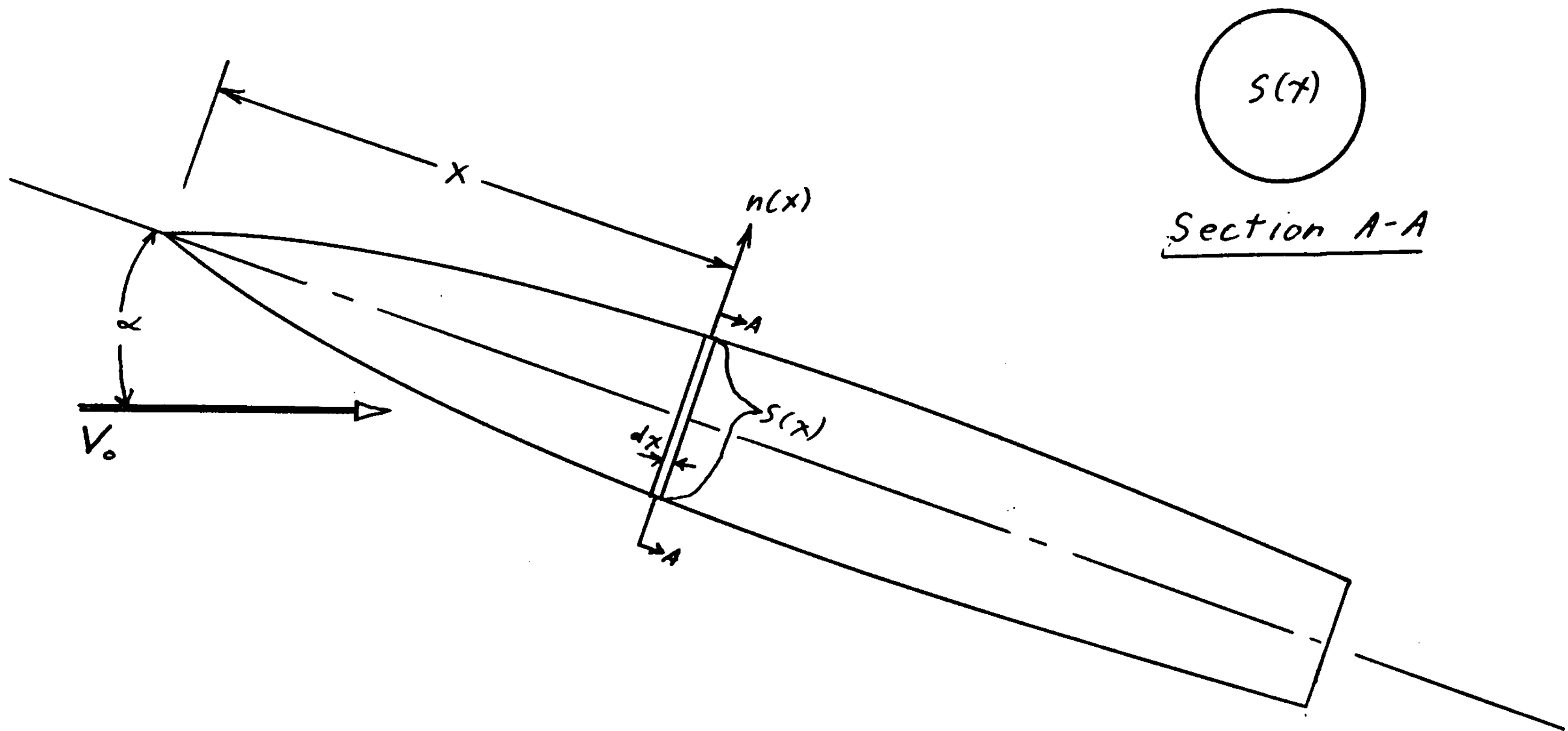


FIGURE 1 BODY STATION PARAMETERS.

but;

$$A = \frac{\pi}{4} d^2$$

therefore;

$$C_N(x) = \frac{8\alpha}{\pi d^2} \frac{\partial S(x)}{\partial x} \qquad (6)$$

By the definition of the normal force coefficient curve slope;

$$C_{N_\alpha}(x) = \left.\frac{\partial C_N}{\partial \alpha}\right|_{\alpha=0} = \frac{8}{\pi d^2} \frac{\partial S(x)}{\partial x} \qquad (7)$$

In order to obtain the total C_{N_α}, Equation 7 is integrated over the length of the body;

$$C_{N_\alpha} = \int_0^L C_{N_\alpha}(x)\,dx = \int_0^L \frac{8}{\pi d^2} \frac{\partial S(x)}{\partial x}\,dx \qquad (8)$$

Since $8/\pi d^2$ is not a function of x;

$$C_{N_\alpha} = \frac{8}{\pi d^2} \int_0^L \frac{\partial S(x)}{\partial x}\,dx \qquad (9)$$

Performing the integration in 9; and noting that the antiderivative of;

$$\frac{\partial S(x)}{\partial x}$$

is;

$$S(x)$$

Then;

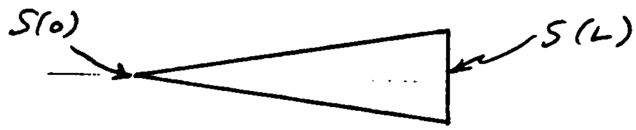
$$C_{N_\alpha} = \frac{8}{\pi d^2}\left[S(L) - S(0)\right] \qquad (10)$$

Immediatly it is noticed that C_{N_α} is independent of the shape of the body as long as the body is such that the integration is valid.

Equation 10 is now applied to the different portions of the body.

Nose

For the nose; $S(0) = 0$



Thus:

$$C_{N_\alpha} = \frac{8}{\pi d^2}\left[S(L) - 0\right] \qquad (11)$$

But;

$$S(L) = \frac{\pi d^2}{4}$$

Thus;

$$\boxed{(C_{N_\alpha})_N = 2} \text{ (per radian)} \qquad (12)$$

This result holds for ogives, cones, or parabolic shapes; as well as any other shape that varies smoothly.

Cylindrical Body

For any cylindrical body; $S(L) = S(0)$

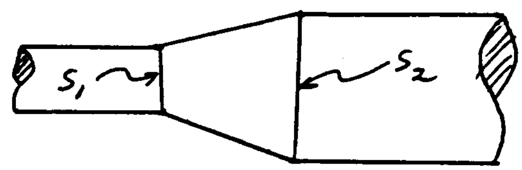
Thus;

$$\boxed{C_{N_\alpha} = 0} \qquad (13)$$

This says that there is no lift on the cylindrical body portions at low angles of attack.

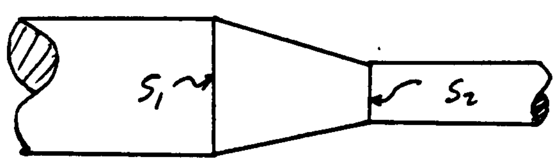
Conical Sholder

Equation 10 is directly applicable to both Conical sholders and boattails



$$\boxed{(C_{N_\alpha})_{cs} = \frac{8}{\pi d^2}(S_2 - S_1)} \qquad (14)$$

Conical Boattail



$$\boxed{(C_{N_\alpha})_{CB} = \frac{8}{\pi d^2}(S_2 - S_1)} \qquad (15)$$

Since S_2 is less than S_1, for a conical boattail, the value of $(C_{N_\alpha})_{CB}$ is negative for angles of attack near zero.

BODY AERODYNAMICS DERIVATIONS

Center of Pressure

General;

By definition, the pitching moment of the local normal aerodynamic force about the front of the body $(x=0)$ is;

$$M(x) = X\,n(x) \qquad (16)$$

Substituting equation 3 into equation 16 =

$$M(x) = \rho V_0^2 \alpha X \frac{\partial S(X)}{\partial X} \qquad (17)$$

By definition of the aerodynamic pitching moment coefficient,

$$C_m(x) = \frac{M(x)}{qAd} = \frac{M(x)}{\frac{1}{2}\rho V_0^2 A} \qquad (18)$$

Substituting equation 17 into equation 18;

$$C_m(x) = \frac{2\alpha X}{Ad}\,\frac{\partial S(x)}{\partial X} \qquad (19)$$

but;

$$A = \frac{\pi}{4} d^2$$

Therefore,

$$C_m(x) = \frac{8\alpha X}{\pi d^3}\,\frac{\partial S(x)}{\partial x} \qquad (20)$$

By the definition of moment coefficient curve slope;

$$C_{m_\alpha}(x) = \left.\frac{\partial C_m(x)}{\partial \alpha}\right|_{\alpha=0} = \frac{8x}{\pi d^3}\frac{\partial S(x)}{\partial x} \qquad (21)$$

In order to obtain the total C_{m_α} equation 21 is integrated over the length of the body;

$$C_{m_\alpha} = \int_0^L \frac{8x}{\pi d^3}\frac{\partial S(x)}{\partial x}\,dx \qquad (22)$$

Since $8/\pi d^3$ is not a function of x ;

$$C_{m_\alpha} = \frac{8}{\pi d^3}\int_0^L x\frac{\partial S(x)}{\partial x}\,dx \qquad (23)$$

Performing the integration in 23 by parts;

$$u = x \qquad dv = \frac{\partial S(x)}{\partial x}dx$$

$$du = dx \qquad v = S(x)$$

$$C_{m_\alpha} = \frac{8}{\pi d^3}\left\{\left[xS(x)\right]_0^L - \int_0^L S(x)\,dx\right\}$$

$$= \frac{8}{\pi d^3}\left\{\left[LS(L) - 0S(0)\right] - \int_0^L S(x)\,dx\right\}$$

$$C_{m_\alpha} = \frac{8}{\pi d^3}\left[LS(L) - \int_0^L S(x)\,dx\right] \qquad (24)$$

By definition, the second term in 24 is the volume of the body;

$$\mathcal{V} = \int_0^L S(x)\,dx \qquad (25)$$

Thus;

$$C_{m_\alpha} = \frac{8}{\pi d^3}\left[L\,S(L) - \mathcal{V}\right] \qquad (26)$$

The center of pressure of the body is defined as;

$$\bar{X} = d\left(\frac{C_{m_\alpha}}{C_{N_\alpha}}\right) \qquad (27)$$

Substituting equations 10 and 26 into equation 27;

$$\bar{X} = \frac{L\,S(L) - \mathcal{V}}{S(L) - S(o)} \qquad (28)$$

Dividing numerator and denominator by $S(L)$;

$$\bar{X} = \frac{L - \mathcal{V}/S(L)}{1 - \frac{S(o)}{S(L)}} \qquad (29)$$

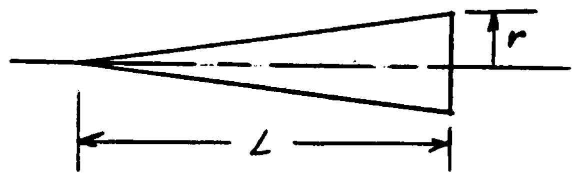
The center of pressure, then, is a definite function of the body shape which determines the volume.

Equation 29 is now applied to the different portions of the body.

Nose

The nose shapes most often used are that of either a cone or an ogive. Thus; $\bar{X}$ is determined for those particular shapes.

Cone



$$v = \frac{\pi}{3} r^2 L = \frac{1}{3} L \, S(L)$$

Thus;

$$v/S(L) = L/3 \qquad (30)$$

also; $S(0) = 0$
thus;

$$\frac{S(0)}{S(L)} = 0 \qquad (31)$$

Therefore;

$$\bar{X} = \frac{L - L/3}{1 - 0}$$

or,

$$\boxed{\bar{X}_N = \frac{2}{3} L} \quad \text{CONE} \qquad (32)$$

Ogive

From reference 2; for a tangent ogive,

$$\frac{v}{4d\,S(L)} = f\left(f^2+\tfrac{1}{4}\right)^2 - \tfrac{1}{3}f^3 - \left(f^2-\tfrac{1}{4}\right)\left(f^2+\tfrac{1}{4}\right)^2 \sin^{-1}\left(\frac{f}{f^2+\tfrac{1}{4}}\right) \qquad (33)$$

where;

$$f = \frac{L}{d} \qquad (34)$$

Again; the denominator is 1, since $S(0) = 0$.
Thus;

$$\bar{X} = L - \frac{v}{S(L)} \qquad (35)$$

Dividing equation 35 by d ;

$$\frac{\bar{X}}{d} = f - \frac{v}{d\,S(L)} \qquad (36)$$

or, substituting equation 33 in equation 36

$$\frac{\bar{X}}{d} = f + 4\left[f\left(f^2+\tfrac{1}{4}\right)^2 + \tfrac{1}{3}f^3 + \left(f^2-\tfrac{1}{4}\right)\left(f^2+\tfrac{1}{4}\right)^2 \sin^{-1}\left(\frac{f}{f^2+\frac{1}{4}}\right)\right] \qquad (37)$$

$$f + 4\left[-f\left(f^2+\tfrac{1}{4}\right)^2 - \tfrac{1}{3}f^3 - \left(f^2-\tfrac{1}{4}\right)\left(f^2+\tfrac{1}{4}\right)^2 \sin^{-1}\left(\frac{f}{f^2+\frac{1}{4}}\right)\right]$$

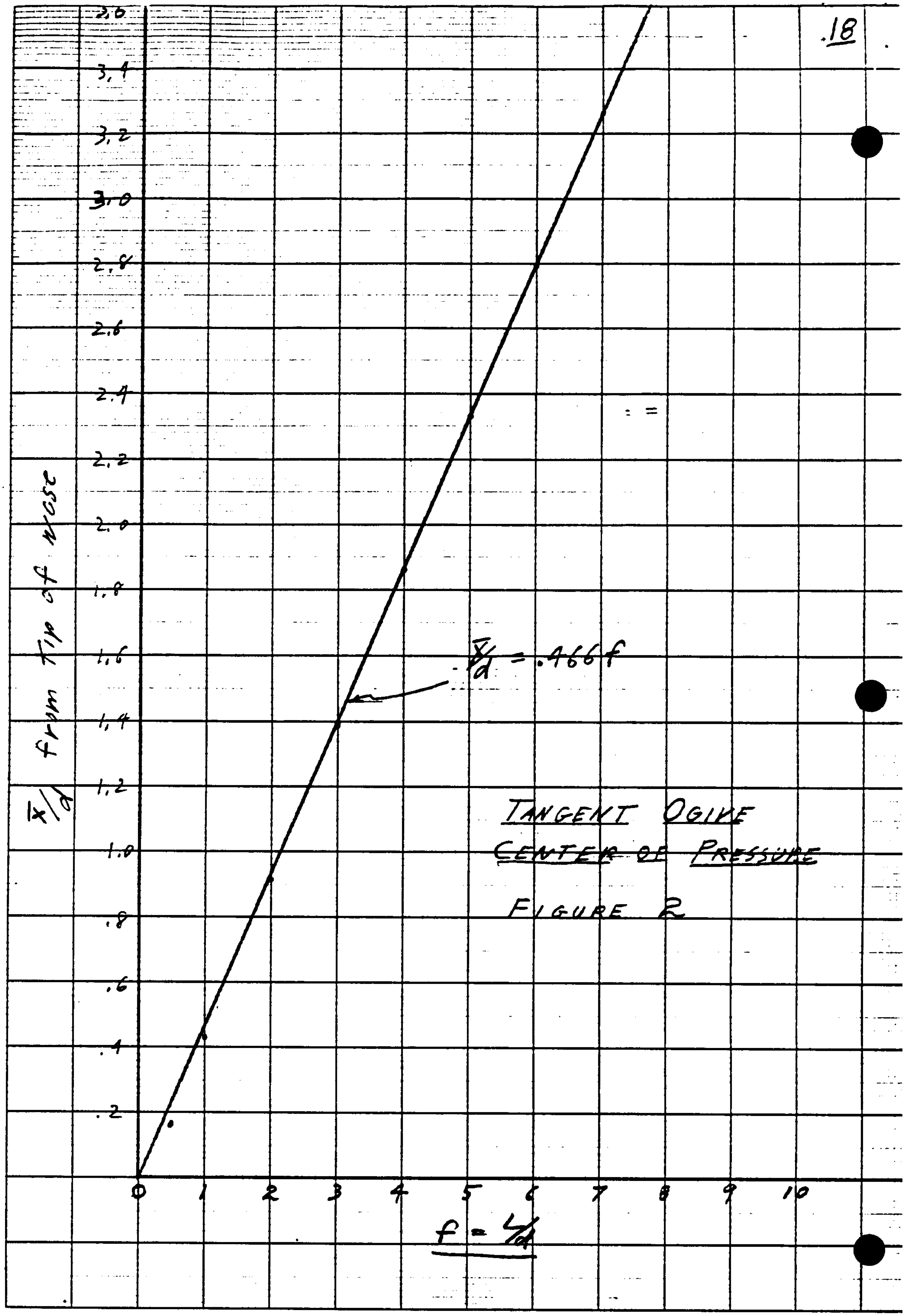
Equation 37 is solved numerically and plotted in figure 2 . A computer program, as listed on the next page, was used to do the calculation with extreme accuracy.

As can be seen in figure 2 , the resultant curve is very nearly a straight line. Thus; equation 37 may be approximated very well be the equation of the straight line as long as f is greater than one (1) .

$$\bar{X}/d = .466\,f = .466\,L/d \qquad (38)$$

dividing both sides of equation 38 by d ;

$$\boxed{\bar{X}_N = .466\,L} \quad \text{OGIVE} \qquad (39)$$



TANGENT OGIVE

CENTER OF PRESSURE

FIGURE 2

```
C       CENTER OF PRESSURE OF AN OGIVE
        DOUBLE PRECISION A,B,C,D,E,F,G,H,XCP
        WRITE(6,2)
      2 FORMAT(13H1    F      X/D)
        DO 10 I=1,10
        F=I
        A=F*F
        B=A+.25
        C=A-.25
        D=B*B
        E=A*F
        G=F/B
        H=DATAN(DABS(G/DSQRT(1.-G*G)))
        XCP = F + 4.*D*(C*H - F) + 4.*E/3.
     10 WRITE(6,1)F,XCP
      1 FORMAT(1H ,F5.0,F9.3)
        STOP
        END
```

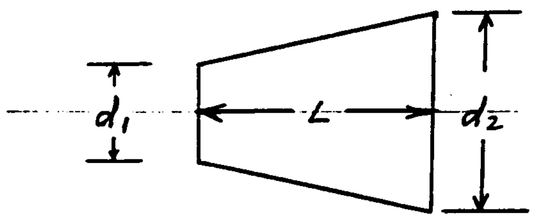
OUTPUT

```
    F      X/D
   1.     0.430
   2.     0.914
   3.     1.387
   4.     1.857
   5.     2.326
   6.     2.794
   7.     3.261
   8.     3.729
   9.     4.196
  10.     4.663
```

Cylindrical Body

Since $C_{N\alpha}=0$ for a cylindrical body, calculation of $\bar{X}$ is not necessary.

Conical Sholder



The volume of a conical frustrum is;

$$v = \frac{\pi L}{12}\left(d_1^{\,2} + d_1 d_2 + d_2^{\,2}\right) \qquad (40)$$

or

$$v = \frac{L}{3}\left(S_1 + S_2\frac{d_1}{d_2} + S_2\right)$$

or

$$v = \frac{L S_2}{3}\left(\frac{S_1}{S_2} + \frac{d_1}{d_2} + 1\right)$$

But, since

$$S_2 = S(L)$$

then,

$$\frac{v}{S(L)} = \frac{L}{3}\left(\frac{S_1}{S_2} + \frac{d_1}{d_2} + 1\right) \qquad (41)$$

Also,

$$\frac{S_1}{S_2} = \left(\frac{d_1}{d_2}\right)^2.$$

thus,

$$v/S(L) = L/3\left[1 + \frac{d_1}{d_2} + \left(\frac{d_1}{d_2}\right)^2\right] \qquad (42)$$

Substituting equation 42 in equation 29;

$$\bar{X} = \frac{L - L/3\left[1 + \frac{d_1}{d_2} + \left(\frac{d_1}{d_2}\right)^2\right]}{1 - \frac{S(0)}{S(L)}} \qquad (43)$$

Again; noting that

$$\frac{S(0)}{S(L)} = \frac{S_1}{S_2} = \left(\frac{d_1}{d_2}\right)^2$$

and expanding;

$$\bar{X} = \frac{L}{3}\left[\frac{3 - 1 - \frac{d_1}{d_2} - \left(\frac{d_1}{d_2}\right)^2}{1 - (d_1/d_2)^2}\right]$$

$$= \frac{L}{3}\left[\frac{2 - d_1/d_2 - (d_1/d_2)^2}{1 - (d_1/d_2)^2}\right]$$

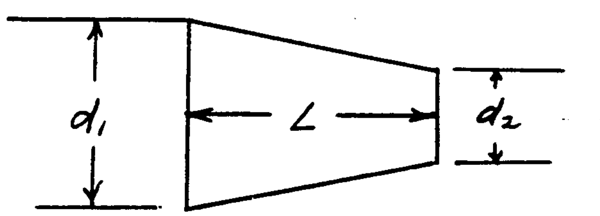
$$= \frac{L}{3}\left[\frac{1 - (d_1/d_2)^2}{1 - (d_1/d_2)^2} + \frac{1 - d_1/d_2}{1 - (d_1/d_2)^2}\right]$$

$$\boxed{\bar{X}_{\substack{cs\\ or\\ cB}} = \frac{L}{3}\left[1 + \frac{1 - d_1/d_2}{1 - (d_1/d_2)^2}\right]}$$ from front of sholder or Boattail (44)

$$\bar{X}_{cs\ or\ cB} = \frac{L}{3}\left(1 + \frac{1}{1 + d_1/d_2}\right)$$ as per [illegible] 8/70

<u>Conical</u> <u>Boattail</u>

Since no distinction as to direction of the conical frustrum was made in deriving equation 44, it holds true also for a frustrum with the dimensions shown;



FIN AERODYNAMICS DERIVATIONS

Normal Force Coefficient Slope

From Reference 1, by a theory of Diederich, C_{N_α} of a finite flat plate is given by;

$$C_{N_\alpha} = \frac{C_{N_{\alpha_0}} F \left(\frac{A_f}{A}\right) \cos\sigma}{2 + F\sqrt{1 + 4/F^2}} \qquad (45)$$

where:

$C_{N_{\alpha_0}}$ = Normal force coefficient slope of a two dimensional airfoil.
F = Diederich's correlation parameter
A_f = Area of one fin

According to Diederich;

$$F \equiv \frac{AR}{\frac{1}{2\pi} C_{N_{\alpha_0}} \cos\sigma} \qquad (46)$$

By the thin airfoil theory of potential flow;

$$C_{N_{\alpha_0}} = 2\pi \qquad (47)$$

Thus;

$$F \equiv \frac{AR}{\cos\sigma} \qquad (48)$$

Substituting equations 47 and 48 into 45;

$$C_{N_\alpha} = \frac{2\pi AR \left(A_f/A\right)}{2 + \frac{AR}{\cos\sigma}\sqrt{1 + \frac{4\cos^2\sigma}{AR^2}}} \qquad (49)$$

Simplifying;

$$C_{N_\alpha} = \frac{2\pi \mathcal{R} \left(A_f/A \right)}{2 + \sqrt{4 + \left(\frac{\mathcal{R}}{\cos \sigma} \right)^2}} \qquad (50)$$

This is C_{N_α} for a single fin.

A typical fin has the geometry shown in figure 3. All fins can be idealized into a fin or a set of fins having straight line edges as shown in figure 3 .

By definition;

$$\mathcal{R} = \frac{2 s^2}{A_f} \qquad (51)$$

Also;

$$A = \frac{\pi d^2}{4} \qquad (52)$$

Substituting 51 and 52 into the numerator of equation 50;

$$2\pi \mathcal{R} \left(A_f/A \right) = 2\cancel{\pi} \left(\frac{2 s^2}{\cancel{A_f}} \right) \left(\frac{\cancel{A_f}}{\frac{\cancel{\pi}}{4} d^2} \right)$$

$$2\pi \mathcal{R} \left(A_f/A \right) = 16 \left(\frac{s}{d} \right)^2 \qquad (53)$$

By trigonometric definition;

$$\cos \sigma = s/l \qquad (54)$$

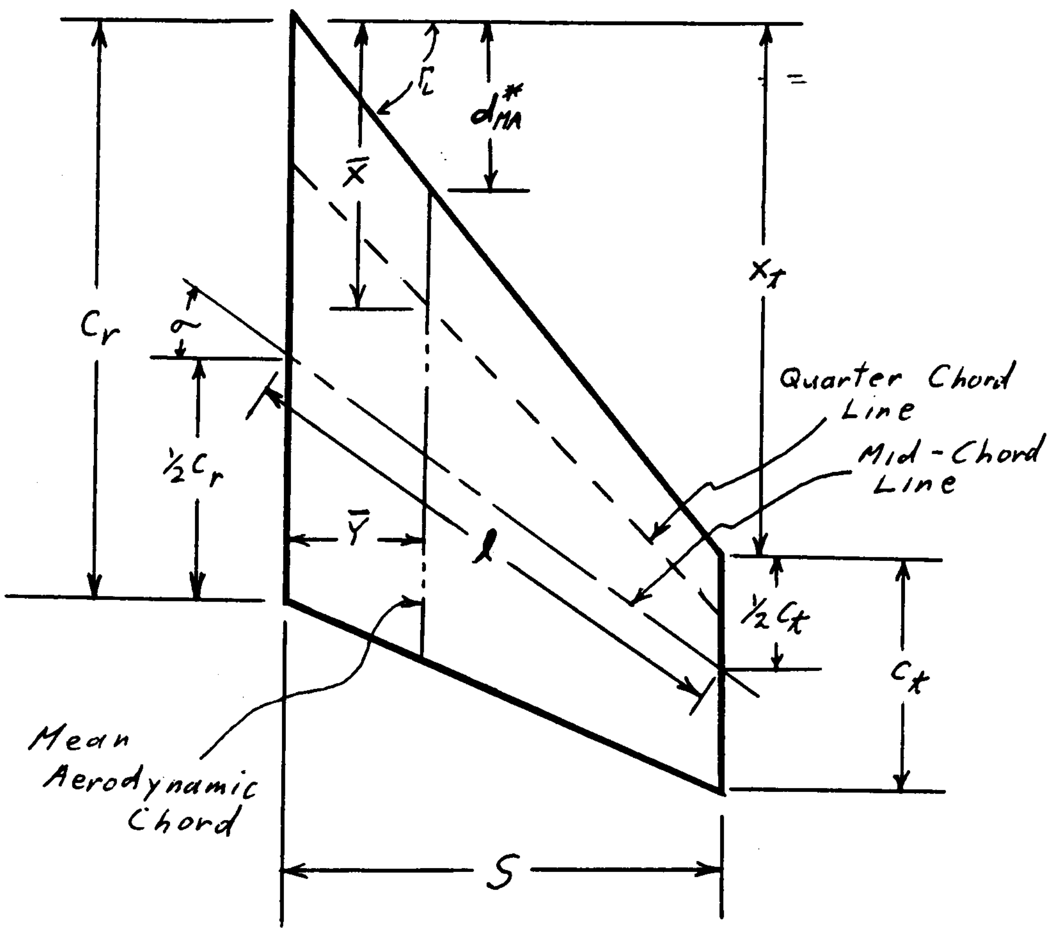


FIGURE 3 - FIN GEOMETRY

Then;

$$\frac{AR}{\cos\sigma} = \frac{2s^{\cancel{2}}}{A_f}\,\frac{\ell}{\cancel{s}} = \frac{2\ell s}{A_f}$$

But; from geometry;

$$A_f = \left(\frac{C_r + C_t}{2}\right)s$$

Therefore;

$$\frac{AR}{\cos\sigma} = \frac{2\ell\cancel{s}}{\left(\frac{C_r+C_t}{2}\right)\cancel{s}}$$

$$\frac{AR}{\cos\sigma} = \frac{4\ell}{C_r + C_t} \qquad (55)$$

Substituting 55 into the denominator of 50;

$$2 + \sqrt{4 + \left(\frac{AR}{\cos\sigma}\right)^2} = 2 + \sqrt{4 + \left(\frac{4\ell}{C_r+C_t}\right)^2}$$

$$= 2 + \sqrt{4 + 4\left(\frac{2\ell}{C_r+C_t}\right)^2}$$

$$2 + \sqrt{4 + \left(\frac{AR}{\cos\sigma}\right)^2} = 2 + 2\sqrt{1 + \left(\frac{2\ell}{C_r+C_t}\right)^2} \qquad (56)$$

Substituting equation 53 and 56 into 50;

$$C_{N_\alpha} = \frac{16\left(\frac{s}{d}\right)^2}{2 + 2\sqrt{1 + \left(\frac{2\ell}{C_r+C_t}\right)^2}}$$

Simplifying;

$$C_{N_\alpha} = \frac{8\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_t}\right)^2}} \qquad (57)$$

Equation 57 gives C_{N_α} for a single fin. A four fin rocket, having two fins in the plane normal to the plane of the angle of attack (see figure 4a) has the C_{N_α} of;

$$\boxed{(C_{N_\alpha})_F = \frac{16\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_t}\right)^2}}} \qquad (58)$$

A three finned rocket has its fins spaced 120° apart. Assuming that the 3 finned rocket flys with one fin in the plane of the angle of attack, with $(C_{N_\alpha})_1 = C_{N_\alpha}$ of one fin; (See figure 4b)

$$C_{N_\alpha} = 2(C_{N_\alpha})_1 \cos 30°$$

$$= 2(C_{N_\alpha})_1 \frac{\sqrt{3}}{2}$$

$$C_{N_\alpha} = \sqrt{3}\,(C_{N_\alpha})_1$$

Thus;

$$(C_{N_\alpha})_F = \frac{8\sqrt{3}\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_r}\right)^2}}$$

or

see addendum 2

$$\boxed{(C_{N_\alpha})_F = \frac{12\,\cancel{13.85}\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{C_r+C_t}\right)^2}}} \qquad (59)$$

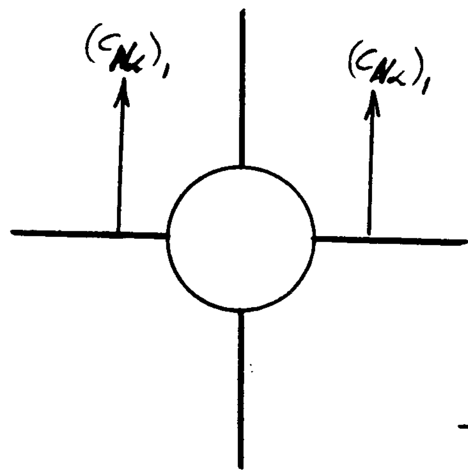


FIGURE 4a
FOUR FINS

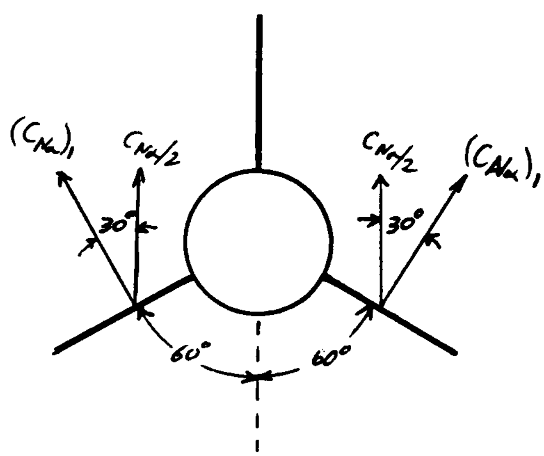


FIGURE 4b
THREE FINS

FIN AERODYNAMICS DERIVATIONS

Center of Pressure

From the potential theory of subsonic flow, the center of pressure of a two dimensional airfoil is located at 1/4 the length of its chord from its leading edge. Thus; on a three dimensional fin, the center of pressure should be located along the quarter chord line.

By definition, the spanwise center of pressure is located along the mean aerodynamic chord. Therefore, by the above argument, the fin center of pressure is located at the intersection of the quarter chord line and the mean aerodynamic chord. (See figure 3)

It remains to determine the length position of the mean aerodynamic chord.

By definition, the mean aerodynamic chord is;

$$C_{MA} = \frac{1}{A_f}\int_0^S c^2\,dy \qquad (60)$$

where; (See figure 5)

A_f = Area of one fin.
S = Semispan of one fin.
c = Generalized chord.
y = Spanwise coordinate.

The generalized Chord is a function of the span. To find this function, a proportionality relation is set up. (See figure 6)

$$\frac{c_r}{L^*} = \frac{c}{L^*-y} = \frac{c_t}{L^*-S} \qquad (61)$$

From the first two terms;

$$c = \frac{c_r\,(L^*-y)}{L^*}$$

$$c = c_r - \frac{y}{L^*}c_r \qquad (62)$$

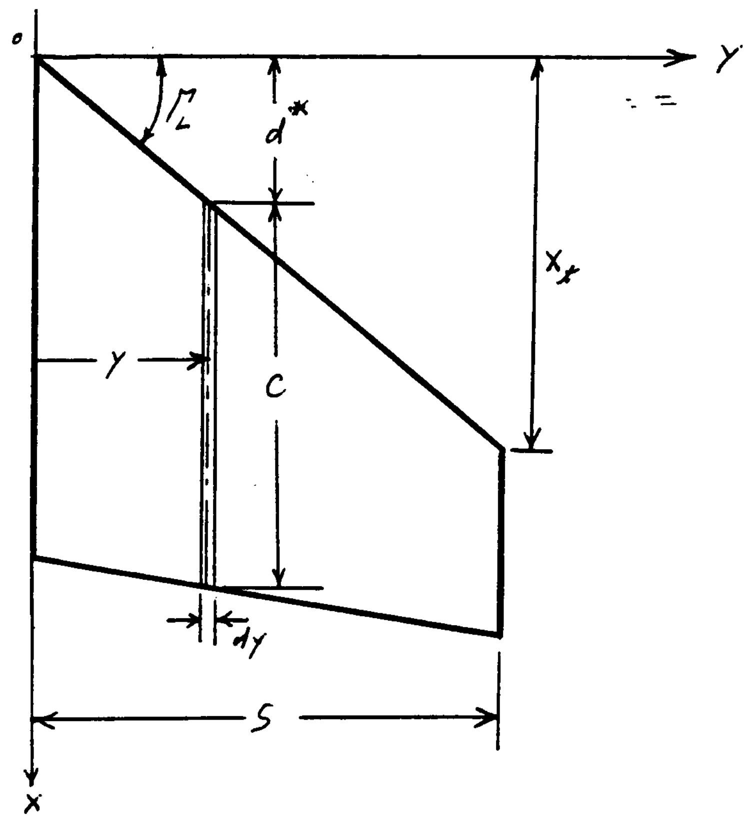


FIGURE 5

COORDINATE SYSTEM FOR THE DETERMINATION OF THE MEAN AERODYNAMIC CHORD

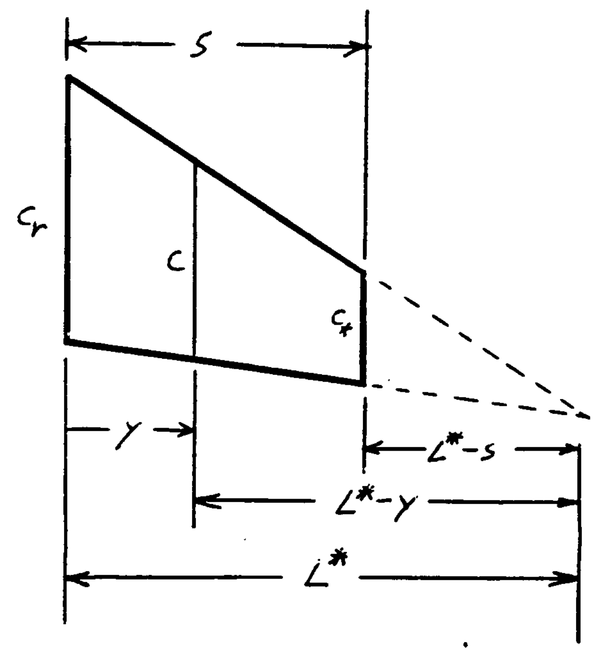


FIGURE 6

TRIANGLE OF PROPORTIONALITY
FOR THE DETERMINATION OF
THE GENERAL CHORD LENGTH

From the first and last terms

$$C_t L^* = C_r L^* - C_r S$$

or

$$L^* (C_r - C_t) = C_r S$$

or

$$L^* (C_r - C_t) = L^* \cancel{C_r} (1 - \lambda) = \cancel{C_r} S$$

thus

$$L^* = \frac{S}{1-\lambda} \qquad (63)$$

substituting 63 into 62

$$C = C_r\left[1 + \left(\frac{\lambda - 1}{S}\right) y\right] \qquad (64)$$

substituting 64 into 60

$$C_{MA} = \frac{1}{A_f}\int_0^S C_r^2 \left[1 + \left(\frac{\lambda-1}{S}\right) y\right]^2 dy$$

Expanding;

$$C_{MA} = \frac{C_r^2}{A_f}\int_0^S \left[1 + 2\left(\frac{\lambda-1}{S}\right) y + \left(\frac{\lambda-1}{S}\right)^2 y^2\right] dy$$

Let; $k = \frac{\lambda - 1}{s}$

$$C_{MA} = \frac{C_r^2}{A_f} \int_0^s \left[1 + 2ky + k^2 y^2\right] dy \qquad (65)$$

Performing the integration;

$$C_{MA} = \frac{C_r^2}{A_f} \left\{ \int_0^s dy + 2k \int_0^s y\,dy + k^2 \int_0^s y^2 dy \right\}$$

$$= \frac{C_r^2}{A_f} \left\{ \left[y\right]_0^s + 2k \left[\frac{y^2}{2}\right]_0^s + k^2 \left[\frac{1}{3} y^3\right]_0^s \right\}$$

$$C_{MA} = \frac{C_r^2}{A_f}\left[s + k s^2 + \frac{1}{3} k^2 s^3\right] \qquad (66)$$

Substitute 66 in 65 and simplifying

$$C_{MA} = \frac{C_r^2 s}{A_f}\left[1 + ks + \frac{1}{3} k^2 s^2\right]$$

$$= \frac{C_r^2 s}{A_f}\left[1 + (\lambda - 1) + \frac{1}{3}(\lambda - 1)^2\right]$$

$$= \frac{C_r^2 s}{A_f}\left[\lambda + \frac{1}{3}(\lambda^2 - 2\lambda + 1)\right]$$

$$C_{MA} = \frac{1}{3} \frac{C_r^2 s}{A_f}\left[\lambda^2 + \lambda + 1\right] \qquad (67)$$

But; by geometry

$$A_f = \frac{1}{2}(C_r + C_t) s \qquad (68)$$

Thus, Substituting 68 in 67;

$$C_{MA} = \frac{2}{3}\frac{C_r^2}{C_r+C_t}\left[\lambda^2+\lambda+1\right]$$

$$= \frac{2}{3}\frac{1}{C_r+C_t}\left[C_t^2+C_rC_t+C_r^2\right]$$

$$= \frac{2}{3}\frac{1}{C_r+C_t}\left[(C_r+C_t)^2-C_rC_t\right]$$

$$C_{MA} = \frac{2}{3}\left[C_r+C_t-\frac{C_rC_t}{C_r+C_t}\right] \qquad (69)$$

It is now necessary to find the spanwise position of C_{MA}. This is done by equating equation 69 and 64 and solving for $\overline{Y}$.

$$\frac{2}{3}\left[C_r+C_t-\frac{C_rC_t}{C_r+C_t}\right] = C_r\left[1+\left(\frac{\lambda-1}{S}\right)\overline{Y}\right]$$

$$= C_r+\left(\frac{C_t-C_r}{S}\right)\overline{Y}$$

Thus;

$$\overline{Y} = \left[\frac{2}{3}C_r+\frac{2}{3}C_t-\frac{2C_rC_t}{3(C_r+C_t)}-C_r\right]\frac{S}{C_t-C_r}$$

$$\bar{Y} = \frac{S}{3(C_t - C_r)}\left[2C_t - C_r - \frac{2C_rC_t}{C_r + C_t}\right]$$

$$= \frac{S}{3(C_t - C_r)(C_r + C_t)}\left[\cancel{2C_rC_t} + 2C_t^2 - C_r^2 - C_rC_t - \cancel{2C_rC_t}\right]$$

$$= \frac{S}{3(C_t - C_r)(C_r + C_t)}\left[2C_t^2 - C_rC_t - C_r^2\right]$$

$$= \frac{S}{3}\left[\frac{(2C_r + C_t)\cancel{(C_t - C_r)}}{\cancel{(C_t - C_r)}(C_r + C_t)}\right]$$

$$\bar{Y} = \frac{S}{3}\,\frac{(C_r + 2C_t)}{(C_r + C_t)} \qquad (70)$$

By trigonometry; (See figure 5)

$$d^*_{MA} = \bar{Y}\tan\Gamma_L \qquad (71)$$

And,

$$\tan\Gamma_L = \frac{x_t}{S} \qquad (72)$$

Thus;

$$d^*_{MA} = \frac{\bar{Y}}{S}\,x_t \qquad (73)$$

Substituting 73 into 70

$$d^*_{MA} = \frac{x_t}{3} \frac{(C_r + 2C_t)}{(C_r + C_t)} \quad (74)$$

From the argument at the beginning of this section;

$$\bar{X} = d + \tfrac{1}{4} C_{MA} \quad (75)$$

Substituting equations 74 and 69 into 75

$$\boxed{\bar{X}_F = \frac{x_t}{3} \frac{(C_r + 2C_t)}{(C_r + C_t)} + \tfrac{1}{6}\left[C_r + C_t - \frac{C_r C_t}{C_r + C_t}\right]} \quad (76a)$$

This $\bar{X}$ is from the leading edge of the root chord. To get the center of pressure of the fins from the nose tip, X_F must be added to $\bar{X}_F$.

X_F = Distance from nose tip to leading edge of fin root chord.

$$(\bar{X})_{T(B)} = X_F + \bar{X}_F \quad (76b)$$

INTERFERENCE EFFECTS

The major interference effects encountered on any rocket are the change of lift of the fin alone when it is brought into the presence of the body and the change of lift on the body between the fins. Reference 3 discusses these effects in detail. They are handled by the use of correction factors which are applied to the fins alone . The values of these factors are shown in figure 7. The plots of interest are underlined in red. In this figure, "S" is actually $(S+r_T)$ in my nomenclature.

$K_{T(B)}$ = Correction factor for the fins in the presence of the body

$K_{B(T)}$ = Correction factor for the body in the presence of the fins.

As can be seen in figure 7, the value of $K_{T(B)}$ is considerably greater than that for $K_{B(T)}$ in the range of $r_T/(S+r_T)$ in which most model rockets fall $(<.4)$. Thus, it is a conservative and reasonable approximation to do two things to simplify the interference calculations.

1) Approximate the $K_{T(B)}$ curve by a straight line. (red line on figure 7)
2) Neglect the influence of $K_{B(T)}$.

In this way;

$$K_{T(B)} = 1 + \frac{r_T}{S+r_T} \qquad (77)$$

Thus;

$$(C_{N_\alpha})_{T(B)} = K_{T(B)} (C_{N_\alpha})_{\text{fins Alone}} \qquad (78)$$

where; $(C_{N_\alpha})_{\text{fins Alone}}$ is obtained from equation 58 or equation 59; and $K_{T(B)}$ comes from equation 77.

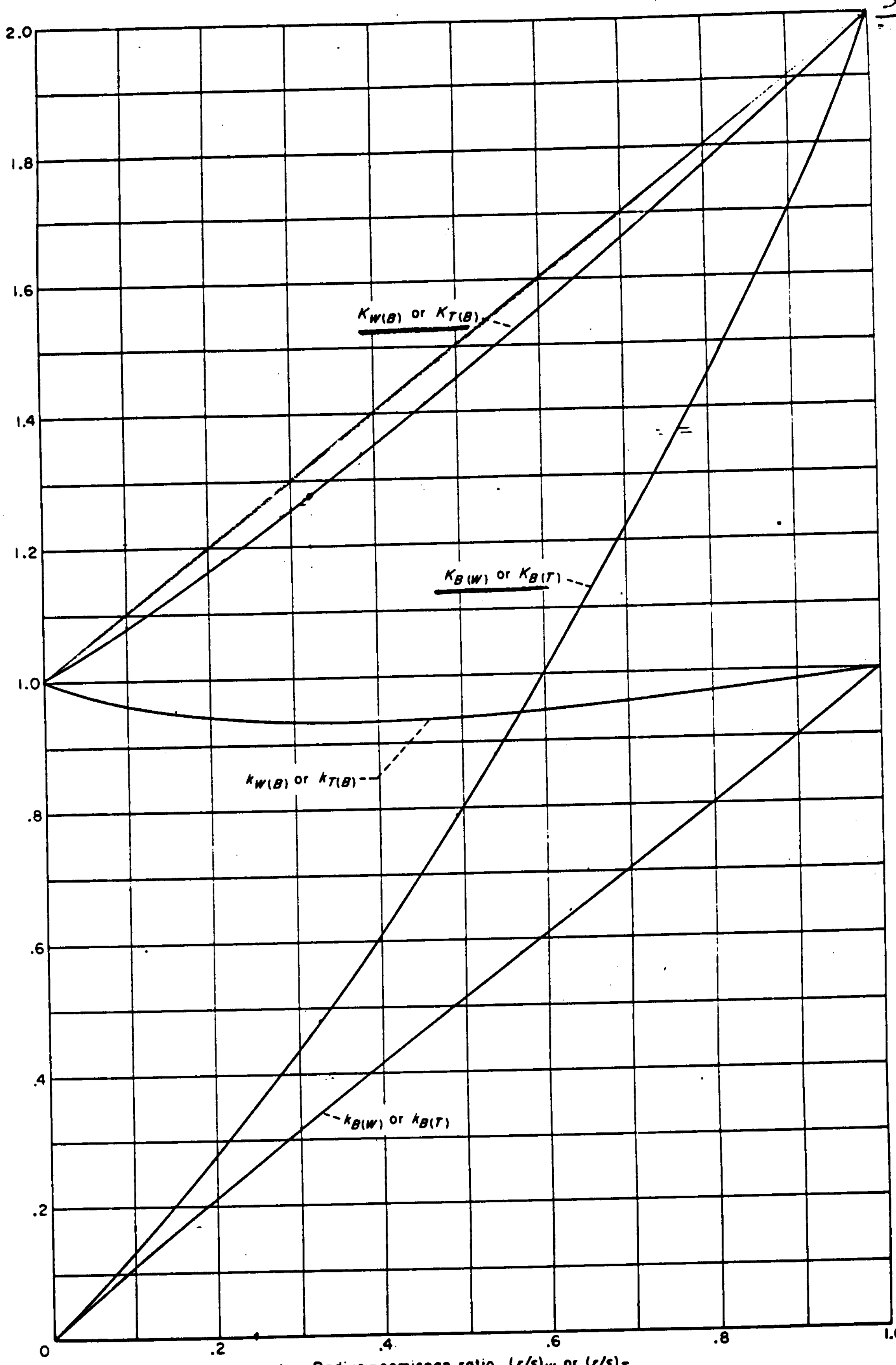


Figure 7 ~~FIGURE 4~~.—Values of lift ratios based on slender-body theory.

COMBINATION CALCULATIONS

The total vehicle $C_{N\alpha}$ is the sum of the $C_{N\alpha}$'s of the individual portions;

$$C_{N\alpha} = (C_{N\alpha})_N + (C_{N\alpha})_{T(B)} + (C_{N\alpha})_{CS} + (C_{N\alpha})_{CB} \qquad (79)$$

The center of pressure is determined by a moment balance about the nose of the rocket.

$$\bar{X} = \frac{(C_{N\alpha})_N \bar{X}_N + (C_{N\alpha})_{T(B)} \bar{X}_{T(B)} + (C_{N\alpha})_{CS} \bar{X}_{CS} + (C_{N\alpha})_{CB} \bar{X}_{CB}}{C_{N\alpha}} \qquad (80)$$

Of course, if there are more than one conical shoulder, conical boattail, and/or fins; these are also included in equations 78 and 79.

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